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Quark matter core in neutron star

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Abstract. The possible phase transition to quark matter in a neutron star is considered using the neutron matter equation of state proposed recently by Canuto, Datta and Kalman which involves spin-2 meson interaction among nucleons. The corresponding equation of state for quark matter is taken from quantum chromodynamics. The Oppenheimer–Volkoff equation is then numerically solved to investigate the stability of a superdense system with quarkion cores.

1. Introduction

There have been a number of investigations (Collins and Perry 1975, Baym and Chin 1976, Chapline and Nauenberg 1977, Baluni 1978a, b, Freedman and McLerran 1977, 1978) to study whether a phase transition between baryon matter and quark matter could take place. The motivation for such investigations arose mainly to satisfactorily answer the interesting question whether or not quarks exist in the core of neutron stars. This was first pointed out by Itoh (1970) who discussed the problem under the assumption that quarks obey para-Fermi statistics. Since then, more realistic approaches have been made among others by Bowers *et al* (1977) and Fechner and Joss (1978). As is obvious, the estimate of critical density at which the phase transition from neutron- to quark-matter might occur depends very much on the particular choice of the equation of state for neutron matter as well as that for quark matter. As for quark matter, the equation of state of a quark gas has been considered both in the phenomenological bag model (Baym and Chin 1976, Chapline and Nauenberg 1976) as well as in the framework of quantum chromodynamics (Collins and Perry 1975, Chapline and Nauenberg 1977). Recently, the quantum chromodynamical (QCD) calculations have been extended (Baluni 1978a, b; Freedman and McLerran 1977, 1978) up to fourth order in the quark–gluon coupling constant to obtain the equation of state for quark matter. A systematic account of the application of QCD at high baryon number densities has been given by Morley and Kisslinger (1979). For an accepted range of the values of the quark mass and quark–gluon interaction parameters they have shown that QCD determines the major characteristics of heavy-mass neutron stars. Unfortunately, for neutron matter at densities appreciably higher than the normal nuclear matter density, the assumption of dominant nucleon–nucleon interaction through the vector–meson exchange (Walecka 1974) is not fully justified, though most conventional theories are based on this kind of idea. Recently, Canuto *et al* (1978) have suggested that at such high density, beyond the normal nuclear density, short-range attractive forces, presumably due to exchange of spin-2 f^0-meson (1260 MeV) among nucleons may play a dominant role. Using an equation of state for neutron matter by

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including spin-2 meson interaction along with vector and scalar mesons, they have considered the possibility of a first-order phase transition between neutron matter (as well as nuclear matter) and quark matter (Canuto *et al* 1979). For the equation of state of quark matter the standard MIT bag model was used. In the present paper we consider the possibility of phase transition between neutron matter and quark matter using QCD for the description of quark matter instead of the bag model and for neutron matter we use the aforementioned realistic equation of state due to Canuto *et al.* Our principal motivation behind this is to investigate the effect of the spin-2 meson coupling constant occurring in the neutron-matter equation of state as well as that of the subtraction constant dependence of chromo fine structure constant in the QCD description of quark matter to obtain the phase transition between neutron matter and quark matter. In § 2 of this paper, we determine the critical densities and transition pressures for a first-order phase transition between neutron matter and quark matter. In § 3, we use these parameters to investigate the possibility of existence of stable superdense neutron stars with quarks at their cores.

2. Baryon–quark phase transition

The full nonlinear theory of an interacting spin-2 field in the Hartree approximation has been worked out in detail by Canuto *et al* (1978) and will not be presented here. We, however, give the relevant expressions for the thermodynamic quantities, namely, the energy density (ϵ) and pressure (P) for neutron matter. These are ($\hbar = c = 1$)

$$\epsilon = \frac{m_f^2 m_N^2}{32\pi f^2}(x^2+3y^2-x^2y^6-3)+\frac{m_N^4}{8\pi^2}\frac{y}{x}J_2+\frac{m_\sigma^2}{2}xy^3\sigma_0^2+\frac{g_v^2 k_F^6}{18\pi^4 x^2 m_v^2}, \tag{1}$$

$$P = -\frac{m_f^2 m_N^2}{32\pi f^2}(x^2+3y^2-x^2y^6-3)+\frac{m_N^4}{24\pi^2}\frac{y}{x}J_3-\frac{m_\sigma^2}{2}xy^3\sigma_0^2+\frac{g_v^2 k_F^6}{18\pi^4 x^2 m_v^2}. \tag{2}$$

The quantities x, y, σ_0, J_1, J_2 and J_3 satisfy the following equations

$$x^4 = x^2y^2+(2f^2m_N^2/3\pi m_f^2)xy(3J_2+J_3)+(16f^2g_v^2k_F^6/9\pi^3 m_v^2 m_f^2 m_N^2), \tag{3}$$

$$x^5y^5 = x^2[xy+(2f^2m_N^2/3\pi m_f^2)(3J_2-2J_3)+(8\pi f^2 m_\sigma^2/m_N^2 m_f^2)x^2y^2\sigma_0^2], \tag{4}$$

$$\sigma_0 = (g_\sigma m_N^2/2\pi^2 xy m_\sigma^2)(m_N-\sigma_0 g_\sigma)J_1, \tag{5}$$

where

$$J_1 = \phi(\phi^2+b^2)^{1/2}-b^2\ln\{[\phi+(\phi^2+b^2)^{1/2}/b\}, \tag{6}$$

$$J_2 = \tfrac{1}{2}b^2J_1+\phi^3(\phi^2+b^2)^{1/2}, \tag{7}$$

$$J_3 = \phi^3(\phi^2+b^2)^{1/2}-\tfrac{3}{2}b^2J_1 \tag{8}$$

with $b^2 = (xy/m_N^2)(m_N-g_\sigma\sigma_0)^2$ and $\phi = k_F/m_N$; k_F being the Fermi momentum, which is related to the neutron number density (n) via the relation

$$n = k_F^3/3\pi^2. \tag{9}$$

In the above equations, m_N is the mass of the neutron and m_σ, m_v, m_f are the masses of the scalar, the vector- and the spin-2 f^0-meson with associated coupling constants g_σ, g_v and f, respectively; σ_0 is the mean Hartree field for scalar meson, while (x, y) gives the mean Hartree field for the spin-2 f^0-meson.

For the quark system, the ground-state equation of state for a gas of massless quarks in second-order perturbation theory in QCD, including the effect of the renormalisation of the quark–gluon coupling constant α_c is given (Chapline and Nauenberg 1977) by

$$\epsilon = \frac{9}{4\pi^2} k_F^4 [1 + (8\alpha_c/3\pi)], \tag{10}$$

$$P = \frac{3}{4\pi^2} k_F^4 [\![1 + (8\alpha_c/3\pi)\{1 - [1/\ln(k_F/\Lambda_F)]\}]\!], \tag{11}$$

where the subtraction dependent chromofine structure constant α_c in a three-flavour quark system is given by

$$\alpha_c = \pi/18 \ln(k_F/\Lambda_F), \tag{12}$$

where k_F again denotes the quark Fermi momentum and Λ_F is the subtraction constant which is expected to lie somewhere in between 200 and 500 MeV—the lower bound of 200 MeV having been set by the consistency requirement that the minimum energy per baryon in the quark matter phase be greater than the nucleon mass (Chapline and Nauenberg 1977). For neutron matter the meson coupling constants are taken to be (Pilkuhn *et al* 1973): $g_\sigma^2/4\pi = 13{\cdot}9$ for σ(700 MeV) and $g_v^2/4\pi = 10{\cdot}0$ for ω(784 MeV). The dimensionless coupling constant f^2 for the f^0 (1260 MeV) meson is not known accurately, the low energy experimentally suggested values being 2·91, 6·55 and 7·44.

Using the expressions for the energy density and pressure for neutron matter and quark matter, one can calculate the chemical potential $\mu = (P + \epsilon)/n$, for each phase of matter as a function of the pressure. The point of intersection of μ versus P curves for the two phases gives the transition pressure for the first-order phase transition between neutron matter and quark matter. We have used the standard Newton-Raphson iteration method to solve equations (3), (4) and (5) for the meson Hartree fields σ_0, (x, y) which are then used in equations (1) and (2) to calculate the energy density and pressure for neutron matter. The corresponding thermodynamic quantities for quark matter are calculated directly from equations (10), (11) and (12) for various values of Λ_F. We find that, for the low-energy experimentally suggested values of f^2 mentioned earlier, there is no phase transition from neutron matter to quark matter with the prescribed range of values of Λ_F. However, taking f^2 as a free parameter, we find that a phase transition can occur only for values of $f^2 \leq 1{\cdot}0$. A similar situation also arose in the calculations of Canuto *et al* (1979) where they used the bag model to describe the quark matter. This, together with plausibility arguments for the appearance of quarks at high density, based on the success of the quark model, led Canuto *et al* to conclude that the coupling constant f^2 might be density dependent, similar to the situation in QCD, namely the coupling constant becoming weak at short interparticle distances (i.e., at high densities). This suggestion can only be acceptable if the underlying spin-2 meson coupling is based on a non-abelian local gauge symmetry. In the present paper, however, we consider f^2 as a free parameter instead of taking it as a density dependent quantity to calculate the equation of state for neutron matter. In figure 1, we display the P versus μ curves for various values of f^2 and Λ_F. We see that the curve with $f^2 = 1{\cdot}0$ intersects only the curve with $\Lambda_F = 200$ MeV. But as one increases Λ_F the value of f^2 has to be decreased in order to achieve a phase transition. For the sake of comparison we also plot the neutron matter curve for $f^2 = 0$ (i.e. with pure scalar and vector meson only). One also notices that for a fixed value of Λ_F, the lower the value of f^2 the lower is the transition pressure for neutron–quark phase transition. The transition densities on

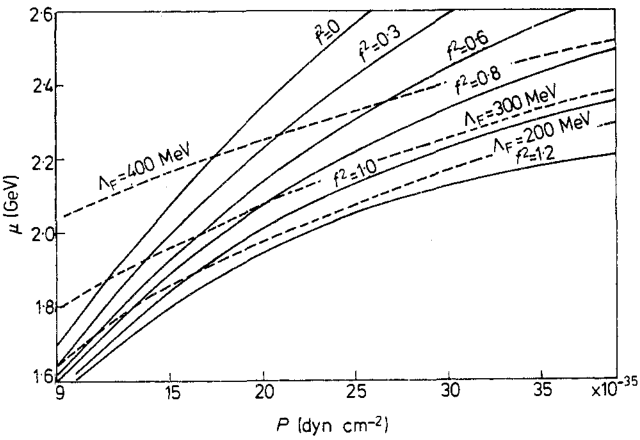

Figure 1. Chemical potential (μ) versus pressure (P) for quark matter (broken curves) and neutron matter (full curves) for various values of the QCD subtraction constant Λ_F and the spin-2 meson coupling constant f^2 (see text).

the neutron matter side as well as those on the quark matter side are determined from the pressure (P) versus density (ρ) curves by using the transition pressures obtained from figure 1. This is illustrated in figure 2 for neutron matter with $f^2 = 0{\cdot}6$ and quark matter with $\Lambda_F = 200$, 300 and 400 MeV. The horizontal broken lines indicate the

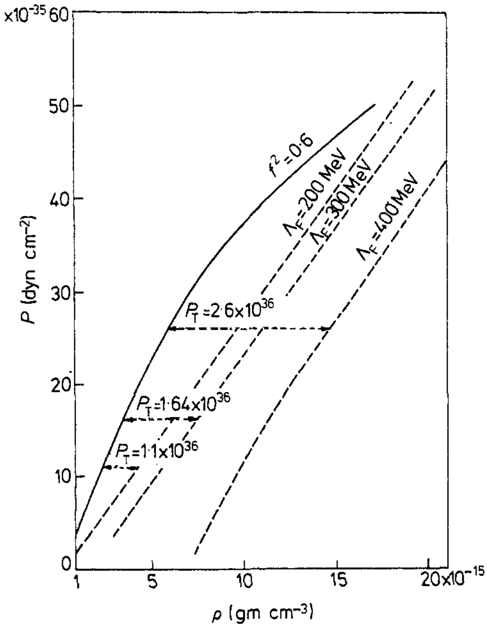

Figure 2. Pressure (P) versus matter density (ρ) for quark matter (broken curves) with $\Lambda_F = 200$, 300 and 400 MeV and neutron matter (full curve) with spin-2 meson coupling constant $f^2 = 0{\cdot}6$. The horizontal double arrowed broken lines indicate the discontinuous rise in density as one passes from neutron matter to quark matter at a first-order phase transition.

discontinuity in density as one passes from neutron matter to quark matter. In table 1, we summarise the results for various transition pressures and densities as obtained from figures 1 and 2. In the next section, we proceed to construct quark star models and investigate their stability by using the phase transition parameters obtained above.

Table 1. Properties of the neutron–quark phase transition. P_T is the transition pressure and ρ_N and ρ_Q denote the matter densities on the neutron matter side and quark matter side of the transition, respectively. The equation of state for neutron matter and quark matter are characterised by the values of the spin-2 meson coupling constant f^2 and the QCD subtraction constant Λ_F, respectively.

f^2	Λ_F (MeV)	P_T (10^{35} dyn cm^{-2})	ρ_N (10^{15} gm cm^{-3})	ρ_Q (10^{15} gm cm^{-3})
	200	7·6	1·5	3·1
0	300	11·8	2·0	5·85
	400	17·4	2·72	11·8
	500	25·8	3·8	23·6
	200	9·2	1·8	3·7
	300	13·8	2·55	6·6
0·3	400	21·0	3·85	13·0
	500	32·4	6·20	25·5
	200	11·0	2·34	4·3
0·6	300	16·4	3·45	7·55
	400	26·0	5·95	14·7
	500	49·7	16·9	31·3
	200	12·3	2·80	4·8
0·8	300	20·9	5·18	9·15
	400	—	—	—
	500	—	—	—
	200	16·1	4·18	6·10
1·0	300	—	—	—
	400	—	—	—
	500	—	—	—

3. Quark star models

Quark star phenomenology with central densities $\simeq 10^{16}$ gm cm^{-3} and beyond has previously been considered by Bowers *et al* (1977), Keister and Kisslinger (1976), Królak *et al* (1978), Fechner and Joss (1978) and others. In order to obtain the mass (M) and radius (R) of a superdense equilibrium configuration, one numerically integrates the Oppenheimer–Volkoff equation of relativistic hydrostatic equilibrium, namely

$$\begin{aligned} dP/dR &= -\frac{G}{c^4}(P+\rho c^2)(Mc^2+4\pi R^3 P)(R^2-2GMR/c^2)^{-1} \\ dM/dR &= 4\pi R^2 \rho, \end{aligned} \tag{13}$$

starting with a given value of the density $\rho = \rho_0$ at the centre up to the point of vanishing pressure at the surface of the dense system. In doing this we use the neutron matter equation of state only up to a certain critical density ρ_c at which a first-order phase transition between neutron matter and quark matter sets in; henceforth for densities higher than ρ_c we use the QCD equation of state for quark matter. For densities less than the normal nuclear density $1{\cdot}6\times10^{14}$ gm cm^{-3}, we use a combination of various equations of state as numerically listed down in detail by Baym *et al* (1971).

To see whether the quarkion configurations could be stable ones, we have obtained the masses and radii for the equilibrium configurations by taking various values of $f^2 \leqslant 1{\cdot}0$ for which quarks are likely to appear through the neutron–quark phase transition. In all cases considered, it turns out that the quarkion configurations have masses which are less than the maximum mass of the neutron stars. This is illustrated in figures 3 and 4 where we display the variation of the masses of the equilibrium configurations in relation to their radii (figure 3) and central densities (figure 4) for a particular value of $f^2 = 0{\cdot}6$ used in describing neutron matter. The curves marked I in figure 3 and 4 correspond to the case when we match the quark matter equation of state with $\Lambda_F = 300$ MeV to the neutron matter equation of state with $f^2 = 0{\cdot}6$ at the critical neutron matter density $\rho_N = 3{\cdot}45\times10^{15}$ gm cm^{-3}; while curves II correspond to the case $\Lambda_F = 400$ MeV with the corresponding $\rho_N = 5{\cdot}95\times10^{15}$ gm cm^{-3} (see table 1). The shapes of the curves together with the criteria of stability given by Bardeen *et al* (1966) clearly indicate that irrespective of the choice of the value of Λ_F, the configurations which contain quarks at their cores (i.e., configurations whose central densities are above the neutron–quark phase transition density) are unstable against radial perturbations. This is in agreement with our earlier investigation (Anand *et al* 1979) where the standard neutron matter model of Bethe and Johnson (Malone *et al* 1975) and the neutron–solid model of Pandharipande and Smith (1975) were used. In the present

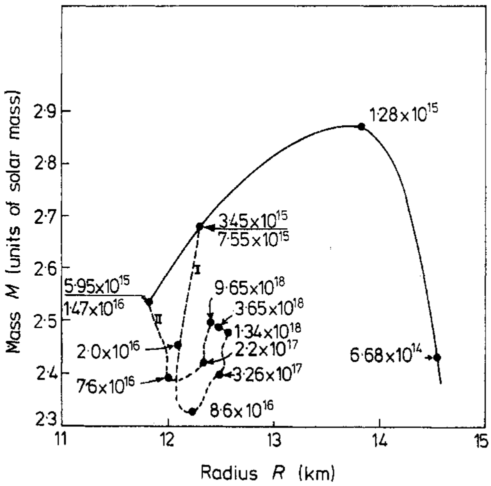

Figure 3. Mass versus radius curves for the equilibrium configurations (parametrised by the central density in units of gm cm^{-3}) obtained by matching the quark matter equation of state with QCD subtraction constant $\Lambda_F = 300$ MeV (curve I) and $\Lambda_F = 400$ MeV (curve II) to neutron matter equation of state with spin-2 meson coupling constant $f^2 = 0{\cdot}6$. The arrows indicate the points at which the configurations with quark matter (broken sections) appear. The numbers above and below these arrows indicate the jump in density at the neutron quark matter phase transition points.

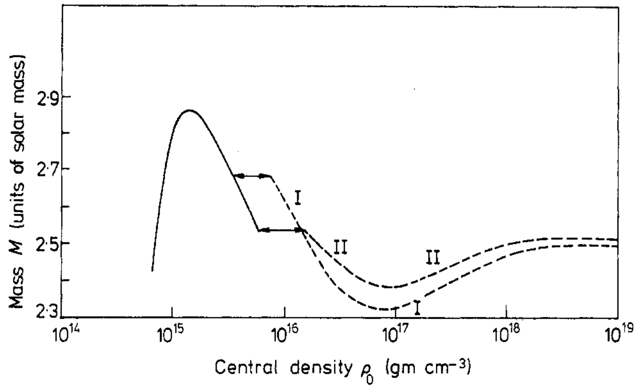

Figure 4. Mass versus central density curves obtained by matching the neutron matter equation of state with $f^2 = 0{\cdot}6$ to the quark matter equation of state with $\Lambda_F = 300$ MeV (curve I) and $\Lambda_F = 400$ MeV (curve II). The dashed portions represent the configurations which contain quarks at their cores. The double arrowed horizontal lines represent the configurations whose central densities lie within the density interval of the neutron–quark phase transition.

case for $f^2 = 0{\cdot}6$ the maximum mass obtained is $M_{\max} \simeq 2{\cdot}87\ M_\odot$ ($M_\odot$ is the solar mass) while for $f^2 = 1{\cdot}0$ we obtain a slightly lower value of $M_{\max} \simeq 2{\cdot}77\ M_\odot$ owing to the equation of state becoming a little softer. On the other hand, a stiffer equation of state with, for example $f^2 = 0{\cdot}3$ gives $M_{\max} \simeq 2{\cdot}94\ M_\odot$. These values are considerably higher than the values of $M_{\max} \simeq 1{\cdot}64\ M_\odot$ in the Bethe–Johnson case and $M_{\max} \simeq 2{\cdot}24\ M_\odot$ in the Pandharipande–Smith case (Anand *et al* 1979). Before concluding we must mention some of the limitations of our present calculations. The neutron matter equation of state used here is taken from the naive relativistic field theoretic treatment of Canuto *et al* (1978), the application of which at the range of densities above 10^{15} gm cm^{-3} may not be adequate. Further, we have also assumed that at densities of the order of 10^{15} gm cm^{-3}, the matter is mainly composed of neutrons only. However, there are calculations which strongly suggest a negative pion condensate (see, for example, Migdal 1978) at half this density. Thus once the stabilising reaction

$$n \leftrightarrow p + e^- + \bar{\nu}$$

is replaced by

$$n \leftrightarrow p + \pi^-$$

the large neutron excess would not occur and the role of nucleon resonances etc cannot be ignored. The effect of these would necessarily alter the baryon equation of state. Hence, on the basis of the model equation of state due to Canuto *et al* and with only the lowest order QCD equation of state for quark matter, it is not possible to ascertain whether a quark star can exist as a stable stellar object.

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